

Introduction to Markov Models
From *Learning Landscape Ecology*, 2nd edition
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OBJECTIVES

Models of landscape change are important tools for understanding the forces that shape landscapes. One motivation for modeling is to examine the implications of extrapolating short-term landscape dynamics over the longer-term. This extrapolation of the status quo can serve as a frame of reference against which to assess alternative management scenarios or test hypotheses. There is a spectrum of ways to consider landscape change, ranging from simple and readily interpretable, to more realistic and less tractable. The goals of this lab are to:

1. Provide an introduction to the mathematics of simple Markov models;
2. Enable students to build a simple model of land use change based on transition probabilities;
3. Explore the process of model creation, verification and validation; and
4. Encourage creative speculation as to how Markov models might be extended to incorporate more complex and realistic mechanisms of and constraints on landscape change.

In this exercise, you will build a simple model of landscape change, evaluate it, and use it as a point of departure to consider more realistic (but more complicated) models. Raster maps of Pacific Northwest forests are compared over three time periods to summarize the rates of transition between cover types. A simple model of landscape change is built from these transition probabilities. Lastly, this model is projected forward in time to verify and validate the model. In order to complete this lab, you will need a PC, the program **markov.exe**, the landscape images (**pnw72.gif**, **pnw85.gif**, **pnw91.gif** and **samp200.gif**) and the accompanying data file (**samp200.xls**) which are located on the in the same webpage directory for this lab.

INTRODUCTION

Perhaps the most fundamental observation on landscape change arises from measurements of the state of a landscape at two time periods. For example, we might have land cover maps classified from satellite images obtained for two dates ten years apart, and note that some of the cells (pixels) changed type over that time interval.

One way to summarize landscape change is to simply tally all the instances, on a cell-by-cell basis, in which a cell changed cover types over that time interval. A concise way of summarizing these tallies is a raw **tally matrix**, which for m cover types is an $m \times m$ matrix. The elements, n_{ij} , of the tally matrix tally the number of cells that changed from type i to type j over a time interval. A raw tally matrix is often converted into proportions by dividing each of the elements by the row total, to generate a **transition matrix P**. The elements, p_{ij} , of the transition matrix P summarize the proportion of cells of each cover type that changed into each other cover type during that time interval. The diagonal elements of this matrix, p_{ii} , are the proportions of cells that did not change.

While there are a variety of approaches to modeling landscape change (see Weinstein and Shugart 1983, Baker 1989, Sklar and Costanza 1991, Mladenoff and Baker 1998 for reviews), many of these begin with a tally matrix or the transition matrix \mathbf{P} . Here, you will examine the simplest of such models based on a transition matrix. This simple model will serve as a point of departure for contemplation of more realistic but more complicated models.

Markov Models

A **first-order Markov model** (Usher 1992) assumes that to predict the state of the system at time $t+1$, one need only know the state of the system at time t . The heart of a Markov model is the **transition matrix \mathbf{P}** , which summarizes the probability that a cell in cover type i will change to cover type j during a single time step. The time step is the interval over which the data were observed to change (*i.e.*, the time interval of the two maps).

Markov models, while simple, have a number of appealing properties. In particular, they can be solved by iteration to project the state of the system. Writing the state of the system as a vector

$$x_t = [x_1 \ x_2 \ x_3 \ \dots] \quad (1)$$

where x_i is the proportion of cells in type i at time t , a Markov model is projected:

$$x_{t+1} = x_t P \quad (2)$$

that is, the state vector post-multiplied by the transition matrix. The next projection, for time $t+2$ is continued:

$$x_{t+2} = x_{t+1} P = x_t P P = x_t P^2 \quad (3)$$

and in general, the state of the system at time $t=t+k$ is given by:

$$x_{t+k} = x_t P^k \quad (4)$$

where x_t is the initial condition of the map. Thus, the model can be projected into the future simply by iterating through the matrix operation (see Exercise 2 for details on how to do this manually).

The steady-state or equilibrium state of the system is given by the eigenvector of the transition matrix; thus, there is a closed-form solution to the model. Recall, the **eigenvector** of the matrix is defined such that the matrix multiplied by the eigenvector yields the vector again:

$$\tilde{x} = \tilde{x} P \quad (5)$$

That is, the system does not change once it reaches this state. There are some computational tricks for estimating steady-state solutions (Usher 1992), or you could use a math package (*e.g.*, Mathematica™, MatLab™) to do this. But for simple models, the solution often converges rapidly and you can estimate the solution simply by projecting the model a few times.

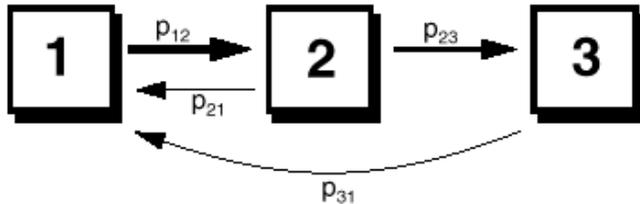


Figure 1

Graphical Representation. The model implied by the transition matrix \mathbf{P} can also be represented as a graph (a “box-and-arrow” diagram). An example with three cover types could be illustrated as in Figure 4.1. Casual inspection of the graph reveals the direction of flow in the system, and suggests a succession from type 1, through type 2 to type 3, with some recycling (possibly a disturbance) to the initial cover type.

Model Projection. To explore a Markov model, it is initialized with a state vector and then projected for one or more time steps. The vector of cover types produced at each iteration is the prediction of overall landscape composition for that timestep. In the following exercises, we will show you how this is accomplished.

EXERCISE

Modeling Landscape Change in the Pacific Northwest, USA

Much debate over the management of Pacific Northwestern (PNW) forests occurred in the early 1990s. The debate centered on the effects of intensive logging on old-growth forests and on old-growth dependent wildlife species such as the northern spotted owl (*Strix occidentalis*) and the marbled murrelet (*Brachyramphus marmoratus*) (Hansen et al. 1991, Ruggiero et al. 1991), primarily on U.S. Forest Service land. Since most of the old-growth on private lands was already gone, most of the old-growth harvested in the 1980s came from these federal lands (Harris 1984, Robbins 1988). By the mid-1990s, harvests were reduced by over 90% on federal lands relative to the peak harvests of the late 1980s (FEMAT 1993, USDA Forest Service and USDI Bureau of Land Management 1994, Marcot and Thomas 1997). A central questions during this debate was, *How long can current rates of harvest be sustained before the old growth is virtually gone?* The reduction in harvest levels in the early 1990s has reduced the urgency of this question for the PNW (Moeur *et al.* 2011), but the relevance of being able to assess trajectories of change is still timely. In PNW forests, the assessment of landscape change has expanded from the initial emphasis on stand-replacing disturbances such as timber harvesting and wildfire (Cohen *et al.* 2002), to partial disturbances (Healey *et al.* 2006) and efforts to quantify variation in the rate of regrowth following disturbance (Schroeder *et al.* 2006, 2007). Initially these analyses relied on the detection of a difference between pairs of Landsat imagery at intervals of approximately five years. More recently, Kennedy *et al.* (2007) have developed an automated approach that is based

on the use of a long time series (>20 years) with an annual time step to describe trajectories of change in forested environments. This approach is based on the notion that many natural and anthropogenic phenomena in forests can be identified on the basis of unique trajectories both before and after the event. This approach is being used to identify a much wider range of disturbance processes, including insect outbreaks, disease and windthrow and also provides a more nuanced description of gradients in disturbance severity (tree retention levels following partial harvest; variation in tree mortality following wildfires).

Study Area. The Oregon Cascades were at the center of this debate over the management of PNW forests. The study area is on federally managed lands where timber harvesting has been conducted using a dispersed (“staggered setting”) system of 10-20 ha clearcut patches. The rate and pattern of these disturbances is somewhat different than those on private lands (Spies et al. 1994) and is quite different than disturbances generated by wildfire during the pre-settlement era (Wallin et al. 1996b).

Spatial data for this area were derived from Landsat Thematic Mapper data using methods outlined in Cohen et al. (1995, 1998, 2002) and Wallin et al. (1996a). Forest cover was classified into 6 approximate age classes (Table 4.1). Images for three time periods are included here: 1972, 1984 and 1991. The images for the different time periods (**pnw72.gif**, **pnw84.gif** and **pnw91.gif**) can be examined by using a web browser (use ‘**File**’, then ‘**Open**’ on your web-browser’s pull-down menu). Yellow areas denote young stands, successively darker greens are older forest, and brown areas are recent clear-cuts. The gray spot in the images is rock. Each image is 500x500 cells (15,625 ha) with a cell size of 25 m.

Table 4.1. Definition of cover types in the Pacific Northwestern forest landscape.

Class	Age (yr)	Cover Type
0	background	(non forest)
1	0-20	recent clearcut
2	21-40	early seral
3	41-80	mid seral
4	81-170	mature
5	> 170	old growth

The file **samp200.gif** shows 200 random locations on the 1972 image. The cover type at each of these locations was tallied at time 1 (1972), 2 (1984), and 3 (1991) on the three maps. This information is tallied in columns of a **primary data matrix** and can be viewed by opening the text file **samp200.dat**.

Next, you will use the cover type data from the 200 sample points to build your own Markov model. In the following exercises you will perform three main steps: model development, model verification, and model validation.

Exercise 1. Model Development

Here, you will use the data from the 200 sample points on the PNW images to calculate transition probabilities for a Markov model. The transition probabilities will be based on landscape change from 1971 to 1984.

1. From the primary data matrix (**samp200.dat**), construct a raw tally matrix that summarizes the number of the 200 cells that underwent a transition from type i to type j during the time period t_1 (1972) to t_2 (1984). Recall that each element, n_{ij} , in the tally matrix is the number of times where a cell changed from type i to type j during the time interval. Enter your results in Table 4.2.
2. Divide each element in the raw tally matrix by its row total to yield a matrix of transition probabilities p_{ij} , the probability (or rate) of change from type i to j . These probabilities are on a 12-year time step (1972-1984). Enter your results in Table 4.3.
3. Next, convert the transition matrix to an annual timestep. This is partly cosmetic (transient dynamics will look smoother), but will also make it possible to reconcile the 12-year timestep of the first period with the 7-year timestep of 1984 to 1991. To convert the transition probability matrix \mathbf{P} to an annual timestep, do the following:
 - (a) Divide each of the off-diagonal elements p_{ij} , $i \neq j$, by 12.
 - (b) Adjust the diagonal elements p_{ii} , to be $1.0 - \sum_j p_{ij}$. In other words, all rows must sum to 1.0. Enter these results in Table 4.4.

In this matrix (Table 4.4), the off-diagonal rates are now annual transitions (probabilities). The diagonal elements are now larger than in Table 4.3 because on an annual time-step fewer of the cells actually change, and again, the rows of the matrix must sum to 1.0.

5. To finish building the model, summarize the state of the map at each timestep. The state of the map is defined by a row vector, the elements of which are the proportion of cells in each cover type in each of the maps. Construct three summary vectors from the primary data matrix (**samp200.dat**). To construct these vectors, simply sum up all of the cells in each state in each of the three years of sampling. Convert these numbers to proportions by dividing each element of the table by its row sum. (These vectors can also be derived from the transition matrices. How?) Enter your results in Table 4.5.

With these state vector tallies and the transition probability matrix \mathbf{P} , you have constructed a simple model of landscape change. All that remains is to evaluate the model. The 1972 data will be used as initial conditions for the model. The second time period (1984) will be used to verify the model. The third time period (1991) will be reserved to validate the model.

Exercise 2. Model Verification: Matrix Projection by Hand

Model verification consists of testing the model against the data used to construct it (Haefner 1996). In this case, the test is of the model projection from 1972 to 1984, compared to the actual data from 1984. Because the model was built from these data, this is *not* an independent test of the model; the model *should*, in fact, match these data. You will verify the model by initializing it with 1972 data and projecting it to 1984, both by hand (here) and by using a computer program MARKOV in Exercise 3. Do the first matrix projection (from 1972 to 1984) by hand following the example below. To do this in one iteration, use the transition probabilities in Table 4.3, which are for a single 12-year timestep.

This example shows a generic matrix projection using only 3 cover types. The first step entails multiplying the transition probability matrix \mathbf{P} by the state-vector.

$$[x_1 \ x_2 \ x_3] \cdot \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} =$$

$$[x_1 p_{11} + x_2 p_{21} + x_3 p_{31}, x_1 p_{12} + x_2 p_{22} + x_3 p_{32}, x_1 p_{13} + x_2 p_{23} + x_3 p_{33}]$$

Note the subscripts to ensure that the proper elements are being used: the inner subscripts must always match (i.e., the subscript on x must match the first subscript on p).

An example using actual data with only three cover types, would look like this:

- (a) Suppose the following transition probability matrix (\mathbf{P}) is used:

$$\mathbf{P} = \begin{bmatrix} 0.90 & 0.10 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.10 & 0.00 & 0.90 \end{bmatrix}$$

- (b) Next, suppose all of the landscape is assigned to cover type 1 to begin:

$$x_0 = [1.00 \ 0.00 \ 0.00]$$

- (c) The first projection, to t_1 is:

$$[1.0 \cdot 0.9 + 0.0 \cdot 0.1 + 0.0 \cdot 0.1, 1.0 \cdot 0.1 + 0.0 \cdot 0.8 + 0.0 \cdot 0.0, 1.0 \cdot 0.0 + 0.0 \cdot 0.1 + 0.0 \cdot 0.9] = [0.9 \ 0.1 \ 0.0]$$

The second projection to t_2 uses the resulting vector $[0.9 \ 0.1 \ 0.0]$ and the original transition probability matrix, and produces:

[0.82 0.17 0.01]

You should verify this by hand. Also, note that these same data are used in the demo data file supplied with the lab (**demo.TXT**).

Exercise 3. Matrix Projection Using the Program MARKOV

Program MARKOV

Not surprisingly, matrix projections are often accomplished with the use of a computer program. Here, you will use a simple fortran program called MARKOV. The program (**markov.exe**) can be run from a DOS prompt or by double-clicking directly on its icon. Demo data are available (**demo.TXT**).

Model Input

MARKOV expects to read a user-provided ASCII data file containing the transition matrix and a vector of initial conditions. For these exercises (with 5 cover types) these must be formatted as follows:

rows 1-5: the elements of the transition matrix (from Table 4.4)

row 6: the initial conditions (the 1972 row from Table 4.5)

The data values themselves can be delimited by spaces, a comma, or tabs, and you should use enough significant digits to minimize round-off error (say, 5 decimal places).

Model Output

The program will report either the timestep at which the solution converged to steady-state, or that the model did not converge during the simulation. In the latter case, you need to rerun the model for a longer time so that it has time to converge. The output written by MARKOV consists of one line per timestep, reporting timestep (in column 1) followed the proportion of the landscape in each cover type at that time (in this case, columns 2-6). The output file from MARKOV is formatted so that it can be imported directly into a spreadsheet or graphics package.

A session with MARKOV is as follows:

```
Project a markov Model?
  Name of file with input data?
  And name of file for output data?
  Number of patch types in model?
  And number of timesteps to project?
  Model failed to converge in 100 timesteps
```

When asked for the number of patch types, this refers to the number of cover types in the model.

Using the the program MARKOV, repeat your model projection from 1972 to 1984 using the following steps.

1. Make an input file (a text file) that includes your matrix and vector data. Format your input data as explained in the *Model Input* section. *NOTE:* you can refer to the file **demo.TXT** for guidance, but remember that these data are for the example with only 3 cover types.
2. Run the program MARKOV.
3. Compare your results from Exercises 1 and 2. Does the model projection reproduce the data used to build the model? If it doesn't reproduce the 1984 data, what might explain the discrepancy?

Exercise 4. Model Validation

Model validation consists of testing a model against data that were *not* used to construct the model (Haefner 1996). This is important as it is an *independent* test of the model.

1. Still using the 1972 data as your initial condition vector, use the program MARKOV to project the model to 1991.
2. Compare the predicted landscape composition to the actual composition tallied from the primary data table. This is a test to **validate** your model. Does the model projection match the 1991 data? If not, what might explain the discrepancy?
3. Continue to project the model into the future, until it converges to a steady-state or until there is less than 10% of the landscape in old-growth forest, whichever comes first. How long will the old-growth last, or when will it equilibrate?

NOTE: You can complete all of these tasks with a single projection of the model. Simply run the model for a very long time (say, 1000 years). If it converges in less time, simply delete the extraneous years from the model output file using a text editor (the output is only interesting while the landscape is changing).

CONCLUSIONS

This concludes the development, verification, and validation of a simple Markov model of landscape change. In some applications, such a simple model is sufficient (e.g., Johnson and Sharpe 1976, Johnson 1977, Hall et al. 1991). But in many cases, this simple model serves as a point of departure for more complicated models (e.g., Turner 1987; Baker 1989; Acevedo et al. 1995, 1996; Wear and Bolstad 1998; Wu and Webster 1998; Hong and Mladenoff 1999a,b; Mladenoff and Baker 1999, Urban et al. 1999). In particular, some consideration of the assumptions and limitations of Markov models can be a useful aid in interpreting the behavior and predictions of other models. These considerations are presented next.

Further Considerations in Modeling Landscape Change

The simple Markov model serves as a useful point of departure for more complicated issues in landscape modeling. Several of these are especially relevant to landscape dynamics.

Stochasticity. Models such as the one you have created are often used to project a map into the future by changing each cell in the map according to the transition probabilities. Because each cell can only change into one other state (a cell can't change fractionally), the state changes must be done probabilistically. This can create some new problems, as the new map is only one of many possible stochastic realizations of a new map (since the map was created probabilistically). Thus, any comparison to a real map would have to be based on a number of replicate maps. Note that, in the limit, the average of a large number of stochastic simulations would be a map of the transition probabilities themselves.

Importance of history. Another issue concerns the assumption that to predict the future state of a system one need only know its current state. In cases where this is true, the process is truly first-order, also known as a Markov chain. In reality, there may be cases where information about additional prior states is needed. These cases would lead to higher-order Markov models (e.g., in a second-order model, one would need to know the state of the system at time t and $t-1$ to predict its state at time $t+1$). Systems with even longer 'memory' would require still higher-order models. For cases where the memory is very long, it might become more convenient to envision (and model) the dynamics in terms of a 'time since' variable, such as 'time since abandonment' or 'time since disturbance,' instead of keeping track of a large number of previous states. Chapter 12: Alternate Stable States uses this approach, using a Time Since Fire variable in a model of fire dynamics.

Stationarity. Because we have three maps, two first-order Markov models could be derived from our study landscape. We could derive transition matrices from 1972 to 1984, and another from 1984 to 1991. There is a formal test for stationarity of these matrices (Usher 1992); nonstationary transition probabilities would vary between time periods. Nonstationary transition matrices would suggest the forces (or rules) governing landscape change were changing over time. Certainly the drivers of landscape change might vary through time in regions with historical variation in socioeconomic drivers, true of most of the United States over the past several decades.

In the case of nonstationary transition rules, two alternatives are possible. In the discrete case, separate transition matrices can be computed for each time period of interest. For example, given a sequence of airphotos taken every 10 years for 50 years, one could derive four separate transition matrices. Each matrix would be used to project from one time period to the next. Alternatively, the transitions could be specified explicitly as functions of time, so that the rules governing landscape change would vary with time. This approach would generate "smoother" dynamics, but would require some sort of curve-fitting for the time functions (as well as the data to support that curve-fitting!).

Spatial dependencies. A fourth complication arises if some of the transitions appear to have spatial dependencies. For example, certain kinds of transitions might tend to occur in certain topographic settings or in certain spatial configurations as defined by a cell's immediate neighbors. These complications drive the modeling approach away from a simple Markov framework, toward models where the transition probabilities depend not only on the current state of the system, but also on some other stated conditions. That is, the transition matrix contains **conditional probabilities** such as "if the cell is type i and meets condition k , then its probability of becoming type j is p_{ijk} ." The condition might relate to site conditions (e.g., soil) or neighborhood effects (e.g., contagious disturbance). It is a relatively straightforward procedure to tally transition matrices as conditional probabilities. One simply constructs a multi-layered tally

matrix analogous to Table 4.4, but incorporating all of the special conditions of interest. Clearly, this can become extremely data-hungry. Again, similar conditional probabilities are examined in Chapter 12.

In any of these more complicated models of landscape change, the ability to solve the model analytically is rapidly lost; thus, complex models must be “solved” by iteration to steady-state (if such a state exists). This trade-off between simplicity and realism is common to all modeling efforts.

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